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Comparison of low Reynolds number k–ε turbulence models in predicting heat transfer rates for pipe flow

C. HRENYA, S. MILLER, T. MALLO and J. SINCLAIR†
Department of Chemical Engineering, Carneigie Mellon University, Pittsburgh, PA 15213, U.S.A.

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Abstract—This paper presents a comparison of nine low Reynolds number k–ε turbulence models in terms of their ability to predict heat transfer rates for pipe flow. The model of Myong and Kasagi is shown to produce excellent agreement with experimental heat transfer data due to the unique ability of this model to accurately capture the radial distribution of the eddy viscosity. © 1998 Elsevier Science Ltd. All rights reserved.

INTRODUCTION

In a recent paper (Hrenya *et al.* [1]), a comparative study of nine different versions of low Reynolds number k–ε turbulence models was carried out. The individual models were evaluated by application to pipe flow over a range of Reynolds numbers. The previous paper extended a comparative study by Patel *et al.* [2] to include the most recent versions of the low Reynolds number models documented in the literature and focused on pipe flow applications which were not considered in Patel *et al.*'s study. The selection of low Reynolds number models for pipe flow was guided by consideration of the results of an investigation carried out by Martinuzzi and Pollard [3]. Their study compared the ability of the high Reynolds number k–ε model, a low Reynolds number k–ε model (Lam and Bremhorst [4]), an algebraic stress model, and a Reynolds stress model to predict developing, turbulent pipe flow over a range of Reynolds numbers. It was found that the predictions from the low Reynolds number model were in the best agreement with the experimental data.

In the low Reynolds number comparative study of Hrenya *et al.* [1], predictions of the mean axial velocity, turbulent kinetic energy, its dissipation rate, Reynolds stress field, eddy viscosity, and skin friction coefficients were compared to experimental data and direct numerical simulation data. It was found that significant qualitative and quantitative differences exist between the model predictions. These relative differences are most apparent in the predictions of the turbulent kinetic energy, its dissipation rate, and the eddy viscosity. The results indicate that one low Reynolds number model, the model proposed by Myong and Kasagi [5], has the best overall performance in predicting turbulent pipe flow.

In the present work, the practical implications of the results from the Hrenya *et al.* [1] study are explored. The particular problem which is considered here is that of heat transfer to gases in turbulent pipe flow. (This investigation is motivated by the present authors' research interest in modeling circulating fluidized bed reactors and is an integral part of that continuing program of work.) Predictions for fully developed Nusselt numbers are made in order to determine the impact of the variations in the fluid dynamic predictions on the corresponding heat transfer rates. For example, while variations in the eddy viscosity profile may not seriously influence the mean velocity profiles predictions, it is anticipated that the accuracy of heat transfer predictions are dependent on such detailed aspects of the flow field. Predictions for the Nusselt number over a range of Reynolds numbers are evaluated based on two correlations, the Gnielinski correlation [6] and the Sleicher–Rouse correlation [7], which afford the best agreement with nearly 800 experimental measurements (Bhatti and Shah [8]). The relative performance of the nine low Reynolds number k–ε models is assessed. The models investigated include those proposed by Chien (CH) [9], Fan *et al.* (FLB) [10], Jones and Launder (JL) [11], Lam and Bremhorst (LB) [4], Launder and Sharma (LS) [12], Lai and So (LSO) [13], Myong and Kasagi (MK) [5], So *et al.* (SZS) [14], and Yang and Shih (YS) [15].

NUMERICAL PROCEDURE

Nusselt numbers for uniform wall temperature were computed with a molecular Prandtl number of 0.7 and a constant turbulent Prandtl number of 0.85 at

† Author to whom correspondence should be addressed. Present address: School of Chemical Engineering, Purdue University, West Lafayette, IN 47907-1283, U.S.A.

NOMENCLATURE

Nu	Nusselt number
Pe_t	turbulent Peclet number, $Pe_t = (\nu_t/\nu)Pr$
Pr	Prandtl number
$Pr_{t\infty}$	turbulent Prandtl number far from the wall
r	radial coordinate
R	pipe radius
Re_{CL}	Reynolds number based on centerline velocity
Re_{AV}	Reynolds number based on average velocity.

Greek symbols

ν	kinematic viscosity
ν_t	eddy viscosity.

Reynolds numbers (based on pipe diameter and centerline velocity) ranging from 10 000 to 50 000. In addition, results were obtained for a turbulent Prandtl number which varied in the near-wall region, increasing steeply as the wall is approached, according to an expression proposed by Kays and Crawford [16]

$$Pr_t = \frac{1}{\dfrac{1}{Pr_{t\infty}} + CPe_t\sqrt{\dfrac{1}{Pr_{t\infty}}} - (CPe_t)^2\left[1-\exp\left(-\dfrac{1}{CPe_t\sqrt{Pr_{t\infty}}}\right)\right]}$$

where $Pr_{t\infty}$ is the value of Pr_t far from the wall ($Pr_t = 0.85$) and C is a constant equal to 0.3. In all of the computations, a non-uniform grid was used in which the number and concentration of points depended on the Reynolds number. Grid densities ranged from 50 to 100 points where at least half of the points were located in the range $r/R > 0.9$.

RESULTS AND DISCUSSION

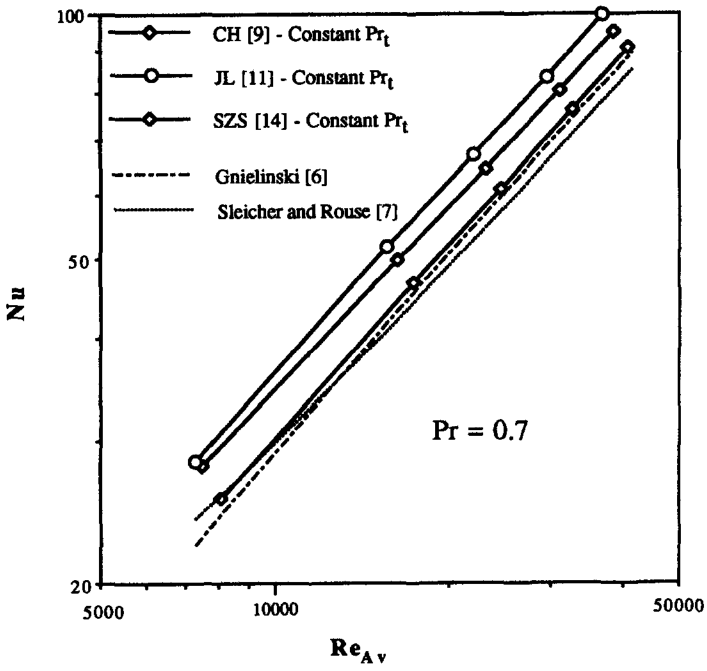
Nusselt numbers computed using each of the nine near-wall turbulence models are compared with the experimental correlations of Gnielinski [6] and Sleicher–Rouse [7] in Figs. 1–3 for both constant and variable Pr_t. The earliest model, JL, renders the poorest agreement with experimental heat transfer rates. Other models which also notably overpredict the heat transfer rate over the range of Reynolds numbers include SZS, YS, LB and LSO. The models of CH, LS and FLB give improved predictions at higher Reynolds numbers. The only model which produces predictions which are in consistent agreement with both correlations is the model of MK. In fact, at a Reynolds number of approximately 15 000 where the discrepancy between the two correlations is at a minimum, the Nusslet number prediction employing the MK model at this Reynolds number is in excellent agreement with both correlations.

From the earlier investigation by Hrenya *et al.* [1] it was observed that only the model of MK is able to predict both the centerline and peak turbulent kinetic energy within 15% of the experimental measurements. Furthermore, only the model of MK is able to generate good quantitative and qualitative agreement with experimental eddy viscosity distributions. The remaining turbulence models predict that the eddy viscosity increases monotonically from the wall and reaches a maximum at the pipe centerline. Hence, the eddy viscosity predictions from the remaining eight models are grossly larger than the experimental values near the pipe center (25%–95% error). The improved eddy viscosity predictions generated with the MK model are the key to accurate predictions of the heat transfer rate. It should be noted, however, that the maximum deviation in the Nusselt number predictions from either experimental correlation with the remaining near-wall turbulence models is 18% (with the JL turbulence model).

The slopes of the heat transfer predictions on the log Nusselt vs log Reynolds plot were computed, along with the slopes generated by the correlations for $Pr = 0.7$. Over the range of Reynolds numbers investigated, the *Nu–Re* number curves were linear (correlation coefficient greater than 0.9999 in all cases) for the near-wall turbulence model predictions and for the correlations. The value for the slope of the *Nu–Re* curve using the Gnielinski correlation [6] was 0.80; the value for the slope using the Sleicher–Rouse correlation [7] was 0.73. The predicted slopes of the *Nu–Re* curves from all of the models fell within this range, except for the model of FLB which overpredicted the slope by 1%.

CONCLUSIONS

Nine low Reynolds number turbulence models have been evaluated in terms of their ability to predict heat transfer to gases flowing in tubes. While large inaccuracies (25%–95% error) were observed in the predictions of the eddy viscosity profiles with all of the models but MK (Hrenya *et al.* [1]), deviations in the Nusselt number predictions from the experimental correlations were less than 18% when a variable turbulent Prandtl number was applied. Nusselt number calculations using a variable turbulent Prandtl num-



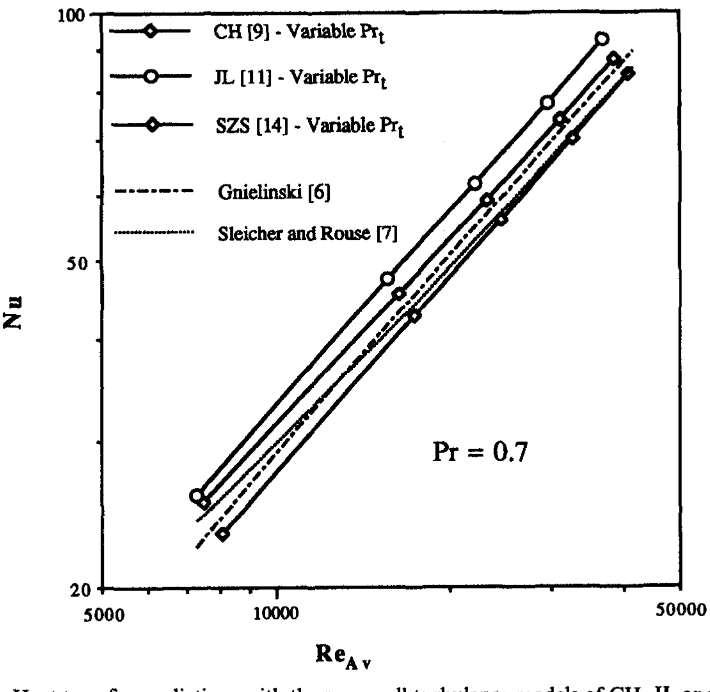


Fig. 1. Heat transfer predictions with the near-wall turbulence models of CH, JL and SZS.

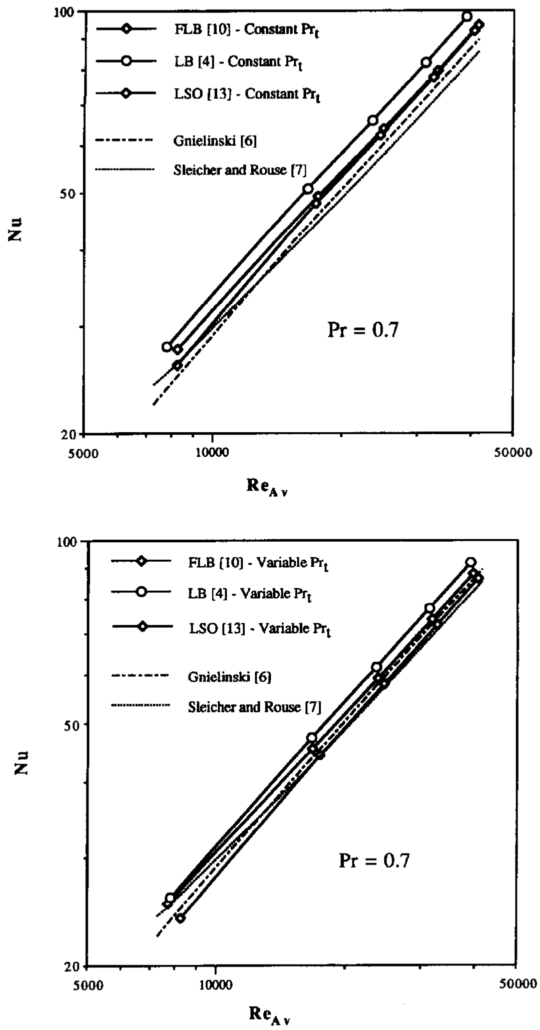


Fig. 2. Heat transfer predictions with the near-wall turbulence models of FLB, LB and LSO.

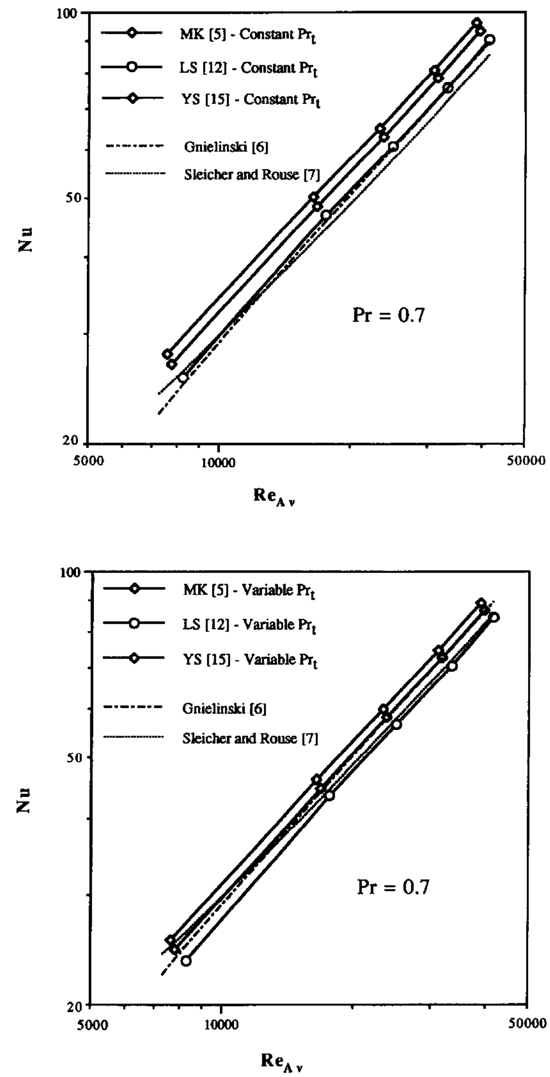


Fig. 3. Heat transfer predictions with the near-wall turbulence models of MK, LS and YS.

ber and the accurate near-wall turbulence model of MK showed excellent agreement with the experimental data over the range of Reynolds numbers studied.

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